Paper Reference(s)

## 6664/01

## Edexcel GCE

## Core Mathematics C2

 Silver Level 55Time: 1 hour 30 minutes
$\frac{\text { Materials required for examination }}{\text { Mathematical Formulae (Green) }} \quad \frac{\text { Items included with question papers }}{\mathrm{Nil}}$

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

## Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C2), the paper reference (6664), your surname, initials and signature.

## Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75 .

## Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

Suggested grade boundaries for this paper:

| A* | A | B | C | D | E |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 70 | 61 | 52 | 44 | 36 | 28 |

1. Find the first 3 terms, in ascending powers of $x$, of the binomial expansion of

$$
(3-x)^{6}
$$

and simplify each term.
2.

$$
y=\frac{5}{\left(x^{2}+1\right)} .
$$

(a) Copy and complete the table below, giving the missing value of $y$ to 3 decimal places.

| $x$ | 0 | 0.5 | 1 | 1.5 | 2 | 2.5 | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | 5 | 4 | 2.5 |  | 1 | 0.690 | 0.5 |



Figure 1
Figure 1 shows the region $R$ which is bounded by the curve with equation $y=\frac{5}{\left(x^{2}+1\right)}$, the $x$-axis and the lines $x=0$ and $x=3$.
(b) Use the trapezium rule, with all the values of $y$ from your table, to find an approximate value for the area of $R$.
(c) Use your answer to part (b) to find an approximate value for

$$
\int_{0}^{3} 4+\frac{5}{\left(x^{2}+1\right)} \mathrm{d} x
$$

giving your answer to 2 decimal places.
3. (i) Solve, for $-\pi<\theta \leq \pi$,

$$
1-2 \cos \left(\theta-\frac{\pi}{5}\right)=0
$$

giving your answers in terms of $\pi$.
(ii) Solve, for $0 \leq x<360^{\circ}$,

$$
4 \cos ^{2} x+7 \sin x-2=0
$$

giving your answers to one decimal place.
(Solutions based entirely on graphical or numerical methods are not acceptable.)
4.

$$
f(x)=x^{4}+5 x^{3}+a x+b
$$

where $a$ and $b$ are constants.
The remainder when $f(x)$ is divided by $(x-2)$ is equal to the remainder when $\mathrm{f}(x)$ is divided by $(x+1)$.
(a) Find the value of $a$.

Given that $(x+3)$ is a factor of $f(x)$,
(b) find the value of $b$.

January 2009
5. (i) Use logarithms to solve the equation $8^{2 x+1}=24$, giving your answer to 3 decimal places.
(ii) Find the values of $y$ such that

$$
\begin{equation*}
\log _{2}(11 y-3)-\log _{2} 3-2 \log _{2} y=1, \quad y>\frac{3}{11} \tag{6}
\end{equation*}
$$

6. 



Figure 2
Figure 2 shows a flowerbed. Its shape is a quarter of a circle of radius $x$ metres with two equal rectangles attached to it along its radii. Each rectangle has length equal to $x$ metres and width equal to $y$ metres.

Given that the area of the flowerbed is $4 \mathrm{~m}^{2}$,
(a) show that

$$
y=\frac{16-\pi x^{2}}{8 x}
$$

(b) Hence show that the perimeter $P$ metres of the flowerbed is given by the equation

$$
P=\frac{8}{x}+2 x .
$$

(c) Use calculus to find the minimum value of $P$.
(d) Find the width of each rectangle when the perimeter is a minimum.

Give your answer to the nearest centimetre.
(2)

January 2012
7. A geometric series is $a+a r+a r^{2}+\ldots$
(a) Prove that the sum of the first $n$ terms of this series is given by

$$
\begin{equation*}
S_{n}=\frac{a\left(1-r^{n}\right)}{1-r} \tag{4}
\end{equation*}
$$

The third and fifth terms of a geometric series are 5.4 and 1.944 respectively and all the terms in the series are positive.

For this series find,
(b) the common ratio,
(c) the first term,
(d) the sum to infinity.
8.


Figure 3


Figure 4

Figure 3 shows a closed letter box $A B F E H G C D$, which is made to be attached to a wall of a house.

The letter box is a right prism of length $y \mathrm{~cm}$ as shown in Figure 3. The base ABFE of the prism is a rectangle. The total surface area of the six faces of the prism is $S \mathrm{~cm}^{2}$.

The cross section $A B C D$ of the letter box is a trapezium with edges of lengths $D A=9 x \mathrm{~cm}$, $A B=4 x \mathrm{~cm}, B C=6 x \mathrm{~cm}$ and $C D=5 x \mathrm{~cm}$ as shown in Figure 4.

The angle $D A B=90^{\circ}$ and the angle $A B C=90^{\circ}$. The volume of the letter box is $9600 \mathrm{~cm}^{3}$.
(a) Show that $y=\frac{320}{x^{2}}$.
(b) Hence show that the surface area of the letter box, $S \mathrm{~cm}^{2}$, is given by $S=60 x^{2}+\frac{7680}{x}$.
(c) Use calculus to find the minimum value of $S$.
(d) Justify, by further differentiation, that the value of $S$ you have found is a minimum.

## END




| Question number | Scheme | Marks |
| :---: | :---: | :---: |
| 5 (i) | $\begin{array}{l\|l} 8^{2 x+1}=24 \\ (2 x+1) \log 8=\log 24 \text { or } & \text { or } 8^{2 x}=3 \text { and so } \\ (2 x+1)=\log _{8} 24 & (2 x) \log 8=\log 3 \text { or }(2 x)=\log _{8} 3 \\ x=\frac{1}{2}\left(\frac{\log 24}{\log 8}-1\right) \text { or } & x=\frac{1}{2}\left(\frac{\log 3}{\log 8}\right) \text { or } x=\frac{1}{2}\left(\log _{8} 3\right) \\ x=\frac{1}{2}\left(\log _{8} 24-1\right) & \text { o.e. } \\ =0.264 & \end{array}$ | M1 dM1 A1 |
| (ii) | $\begin{aligned} & \log _{2}(11 y-3)-\log _{2} 3-2 \log _{2} y=1 \\ & \log _{2}(11 y-3)-\log _{2} 3-\log _{2} y^{2}=1 \\ & \log _{2} \frac{(11 y-3)}{3 y^{2}}=1 \quad \text { or } \quad \log _{2} \frac{(11 y-3)}{y^{2}}=1+\log _{2} 3=2.58496501 \\ & \log _{2} \frac{(11 y-3)}{3 y^{2}}=\log _{2} 2 \text { or } \log _{2} \frac{(11 y-3)}{y^{2}}=\log _{2} 6 \text { (allow awrt } 6 \text { if } \\ & \text { replaced by } 6 \text { later) } \\ & \text { Obtains } 6 y^{2}-11 y+3=0 \text { o.e. i.e. } 6 y^{2}=11 y-3 \text { for example } \\ & \text { Solves quadratic to give } y= \\ & y=\frac{1}{3} \text { and } \frac{3}{2} \text { (need both- one should not be rejected) } \end{aligned}$ | M1 <br> dM1 <br> B1 <br> A1 <br> ddM1 <br> A1 |
|  |  | (6) <br> [9] |




| Question number | Scheme | Marks |
| :---: | :---: | :---: |
| 8 (a) | $\begin{aligned} & \quad \frac{1}{2}(9 x+6 x) 4 x \\ & \text { or } \quad 2 x \times 15 x \\ & \text { or }\left(\frac{1}{2} 4 x \times(9 x-6 x)+6 x \times 4 x\right) \\ & \text { or } \quad 6 x^{2}+24 x^{2} \\ & \text { or }\left(9 x \times 4 x-\frac{1}{2} 4 x \times(9 x-6 x)\right) \\ & \text { or } \quad 36 x^{2}-6 x^{2} \\ & \Rightarrow 30 x^{2} y=9600 \Rightarrow y=\frac{9600}{30 x^{2}} \Rightarrow y=\frac{320}{x^{2}} * \end{aligned}$ | M1A1cso <br> (2) |
| (b) | $\begin{gathered} (S=) \frac{1}{2}(9 x+6 x) 4 x+\frac{1}{2}(9 x+6 x) 4 x+6 x y+9 x y+5 x y+4 x y \\ y=\frac{320}{x^{2}} \Rightarrow(S=) 30 x^{2}+30 x^{2}+24 x\left(\frac{320}{x^{2}}\right) \end{gathered}$ <br> So, $\quad(S=) 60 x^{2}+\frac{7680}{x}$ * | M1A1 <br> M1 $\mathrm{A} 1 * \text { cso }$ <br> (4) |
| (c) | $\frac{\mathrm{d} S}{\mathrm{~d} x}=120 x-7680 x^{-2}\left\{=120 x-\frac{7680}{x^{2}}\right\}$ | M1 |
|  | $\begin{aligned} & 120 x-\frac{7680}{x^{2}}=0 \\ & \Rightarrow x^{3}=\frac{7680}{120} ;=64 \Rightarrow x=4 \\ &\{x=4,\} \quad S=60(4)^{2}+\frac{7680}{4}=2880\left(\mathrm{~cm}^{2}\right) \end{aligned}$ | M1A1cso <br> ddM1 <br> A1 cao and cso |
| (d) | $\begin{aligned} \frac{\mathrm{d}^{2} S}{\mathrm{~d} x^{2}} & =120+\frac{15360}{x^{3}}>0 \\ & \Rightarrow \text { Minimum } \end{aligned}$ <br> Note parts (c) and (d) can be marked together. | M1A1ft <br> (2) <br> [14] |

## Examiner reports

## Question 1

This binomial expansion was answered well, with a majority of the candidates scoring three or four marks. The binomial coefficients were usually correct, though a few used ${ }^{5} C_{r}$ instead of ${ }^{6} C_{r}$.Those using the $(a+b)^{n}$ formula were the most accurate. The majority of errors with that method being with $+/-$ signs: using $x$ instead of $-x,(-x)^{2}$ becoming $-x^{2}$, not simplifying $1458(-x)$ to $-1458 x$ or leaving as $+(-1458 x)$. Attempts to take out the 3 to use the $(1+x)^{n}$ expansion were generally less successful with candidates not raising 3 to a power or not dividing the $x$ term by 3 . There were a number of marks lost by slips such as miscopying 729 as 792 or 726 , or neglecting the $x$ in the second term.

## Question 2

This was another straightforward question with most candidates finding parts (a) and (b) very accessible. Rounding errors in (a) were seen in a small number of cases, but incorrect answers were very rare.

There were some common errors in (b): for example, inclusion of the final $y$ value, 0.5 , in the inner bracket, often outside it as well; incorrect values for $h$, usually $\frac{3}{7}$, but occasionally 5 , or even 6 ; missing external brackets, e.g.: $\frac{1}{2} \times \frac{1}{2}(5+0.5)+2(4+2.5+1.538+1+0.690)$, resulting in a final answer of 20.831 rather than 6.239 . In many cases, candidates wrote a completely correct expression for the area, but their calculation implied they had included the 0.5 ordinate in the internal bracket. Use of individual trapezia was used occasionally; however, use of $x$-instead of $y$-values was seldom seen.

It is noteworthy that, even in a lot of correct answers, the final closing bracket was missing from the working. With the formula in the printed book these errors should not happen. Brackets are an important part of mathematics.

In part (c) many candidates failed to realise that this part related to their previous answer. Instead, a common response was to try and integrate the function, by handling the algebraic fraction incorrectly, e.g. putting $\int_{0}^{3}\left(4+\frac{5}{x^{2}+1}\right) \mathrm{d} x=\int_{0}^{3}\left(4+\frac{5}{x^{2}}+5\right) \mathrm{d} x$.
Some students realised there was a connection between parts (b) and (c) but simply added 4 to their previous answer.

Those who handled this question best showed good understanding of the graph transformation and used geometry to find the area of the added rectangle $(3 \times 4=12)$ and added that to their answer from part (b). Another method which worked successfully was to add 4 to each function value in the table and then to use the trapezium rule again to calculate the area.

## Question 3

This trigonometry question was answered well by many candidates, showing some improvement this year. A fair number gained full marks, with a similar number losing only one or two marks. Unfortunately, however there were some who gained no marks on this question.

Part (i) should have been an easy three marks but a sizeable number of candidates only got the first mark for correctly rearranging the equation to give $\cos \left(\theta-\frac{\pi}{5}\right)=\frac{1}{2}$. A mixture of graphs and CAST diagrams were seen and candidates who used the CAST diagram to find additional solutions usually fared better than those who attempted the graphical method. A small minority of candidates attempted the use of general formulae to find solutions.

Some re-arranged the equation to $\cos \left(\theta-\frac{\pi}{5}\right)=-\frac{1}{2}$ and were unable to gain any further marks. A number of candidates worked in degrees but failed to convert back to radians for the final two marks. Common mistakes included subtracting $\frac{\pi}{5}$ from their principal value instead of adding and many candidates found only one answer, usually $\frac{8 \pi}{15}$.

Part (ii) was answered well with most candidates replacing $\cos ^{2} x$ with $1-\sin ^{2} x$ and proceeding to a correct three term quadratic. They then mostly went on to factorise and solve the quadratic correctly to obtain $\sin x=-\frac{1}{4}$ and $\sin x=2$. Some used the formula and some candidates introduced a dummy variable to replace $\sin x$ in order to solve their quadratic. However, it was not uncommon to see candidates defining $x$ to replace $\sin x$ instead of introducing a different variable and then becoming confused when they had solutions to their quadratics, believing these to be their final answers and so not using arcsin.

Many had difficulty finding the required range of solutions which were in the third and fourth quadrants. A significant number found one solution but had an incorrect one as the second. As always, some lost a mark for including extra solutions inside the range. Rounding errors also lost the final accuracy mark in some cases.

## Question 4

In part (a) most who used the remainder theorem correctly used $f(2)$ and $f(-1)$ and scored M1A1 usually for $16+40+2 \mathrm{a}+\mathrm{b}$, the $(-1)^{\wedge} 4$ often causing problems. A large number of candidates then mistakenly equated each to zero and solved the equations simultaneously, obtaining $a=-20$ and ignoring $b=-16$ so that they could go on in $(b)$ to use $f(-3)=0$ to obtain $b=-6$.

Those who equated $f(2)$ to $f(-1)$, as required, usually completed to find a although there were many careless errors here. Some candidates worked with $f(2)-f(-1)$ and then equated to zero but not always very clearly.

## Question 5

In part (i) most candidates gained the first mark by stating $(2 x+1) \log 8=\log 24$ and then successfully rearranged to obtain the printed answer $x=0.264$. Rearranging the equation $2 x+1=1.528 \ldots$ to make $x$ the subject proved difficult for some candidates as they added 1 to both sides rather than subtracted, or they divided through by 2 before subtracting 1 .

A common error was following $(2 x+1) \log 8=\log 24$ with $(2 x+1)=\log (24 / 8)$ instead of $(2 x+1)=\log 24 / \log 8$.

A more unusual method seen was using $8^{2 x+1}=8^{2 x} \times 8$ followed by division through by 8 before taking logs of both sides of the equation. A few candidates continued to change the 8 to $2^{3}$, proceeding to $2^{6 x+3}$ before taking logs of both sides of the equation.

The question had asked for the use of logs, so an answer with no working gained no credit here.

In Part (ii) the majority of candidates gained the first mark by replacing $2 \log _{2} y$ with $\log _{2} y^{2}$ and, at some point in their working, using $\log _{2} 2$ or $2^{1}=2$. There was more success than in previous sessions in combining logs correctly, but difficulties arose where candidates created a triple fraction.

Candidates who rearranged the equation so that $\log _{2} y^{2}$ was on the right hand side didn't produce the triple fraction so they tended to progress to the correct quadratic. Some checking of fraction work was in evidence and the candidates who did this, generally reached the correct quadratic.

Those who combined terms correctly and arrived at $2^{2.584962501 \ldots}$ almost always changed to 6 and proceeded to the correct quadratic and final solutions.

A few successful candidates changed $-2 \log _{2} y$ to $+\log _{2} y^{-2}$ before collecting terms and obtaining correct solutions.

Where a quadratic in $y$ was obtained following reasonable log work, most candidates were able to use a correct method to solve it. Of those who obtained the correct quadratic, almost all candidates used the quadratic formula to solve and find both solutions $y=\frac{3}{2}$ and $y=\frac{1}{3}$, but some then rejected $y=\frac{1}{3}$, typically stating $\frac{1}{3}<\frac{3}{11}$ as their justification.

## Question 6

About $25 \%$ of the candidates achieved full marks or lost just one mark. The lost mark was usually forgetting to evaluate the minimum perimeter, or making errors giving the final answer to the nearest centimetre

Part (a): Most candidates had some idea how to form an equation for the area although an incorrect fraction of the circle was often seen. The algebra that followed was not always reliable although those that multiplied all the terms by 4 to start with had greater success than the others. Several candidates were clearly uncomfortable with dealing with double fractions.

Part (b): This was the most challenging part of the question, with candidates frequently getting the wrong coefficients for at least one of $x, y$ or $x^{2}$. Most knew they had to substitute the value of $y$ from (a) but sometimes the expression had been changed before it was substituted. A common error was to multiply all terms by an integer (usually 2 ) to remove the fraction, but not apply this to P (on the left hand side of the equation). The algebraic rearrangement that followed the substitution was often laboured and frequently inaccurate.

Part (c): This part of the question was generally well done and there were many responses where it was the only part that was awarded marks. A large number of candidates failed to use their value of $x$ to find P (this was sometimes the only mark lost on the paper). Candidates generally appeared to have little difficulty with the differentiation and the subsequent rearrangement of their equation to find $x$. There was some reluctance to include a statement indicating that $\frac{\mathrm{d} P}{\mathrm{~d} x}=0$ in forming their equation for finding $x$.

Part (d): Some candidates thought that they had to find $P$ here and failed to find $y$. Of those that did calculate $y$, most used the formula from part (a) and were generally successful in getting to $0.21 \ldots$. . It was not unusual for $P$ and $x$ to be substituted into the original expression for $P$, which was then rearranged, sometimes successfully, to find $y$.

However, there was great confusion with units and 0.21 cm (often rounded to 0 or 1) was probably the most commonly seen answer. Those who converted their answer to cm were usually successful with 21 cm , although 2.1 cm or 210 cm were occasionally seen as were 21.5 cm and 22 cm . The alternative correct answer of 0.21 m was relatively rare.

## Question 7

Most students should have learned a proof of the sum to $n$ terms of a geometric progression, but only a minority were able to construct a complete proof in part (a). Unfortunately a significant proportion of candidates were unable to deal with a finite series of unknown length. The majority of students attempted the traditional method of subtracting expressions for $S_{n}$ and $r S_{n}$ and then factorising and dividing. Marks were not given here for factorisation that clearly did not follow from their own series. Some students attempted a proof by induction (from the FP1 specification) but this was rare. Some even tried to use the method for proving the sum of an arithmetic series. Teachers need to emphasise that these are easy marks and students must learn how to present and reproduce these formal proofs.

In part (b) most students were able to find the common ratio but there were some serious errors such as subtracting the 1.944 from the 5.4 instead of finding their ratio, or using $r=0.36$ instead of 0.6 . Most then went on successfully to find the first term in part (c) and the sum to infinity in part (d). Those who struggled with part (b) generally went on to gain a follow through mark in part (c), and a method mark in part (d) although this mark was not given if $|r|>1$. Once again, a few treated this as an arithmetic series.

The most common score was 0 in part (a) and full marks in the rest of the question. About $20 \%$ achieved full marks and about $40 \%$ achieved 7 out of the 11 marks while about $15 \%$ achieved no marks at all.

## Question 8

This question involved several different areas of work, area, volume, algebraic manipulation and calculus, and although a significant number of students produced clear and wellstructured solutions, this proved a taxing question for many students.

Part (a) was found to be challenging, with many students struggling to find the volume of the prism despite there being several possible methods. It was common to see $30 x^{2} y=9600$ derived with unconvincing or incorrect working, despite often being able to find the area of a trapezium correctly in part (b). Some students, realising that $30 x^{2}$ must be the area of the trapezium, just used $6 x \times 5 x$. In part (b) there were many concise, correct and clearly set out solutions but it was very common to see several attempts and much crossing out, and extra terms slotted in at a late stage, presumably influenced by the required expression being given. Common errors included not finding areas of all 6 faces, finding too many areas, and combining dimensionally incorrect areas.

Almost all students who attempted the question gained marks in part (c) with most differentiating at least one term correctly and many achieving the correct $S$ and setting it equal to 0 . Many, though, found it difficult to manipulate their equation correctly and of those who reached $x^{3}=64$ many did not find $x=4$. Common answers were $x= \pm 4$ or $x=8$. Many lost the last two marks by not realising that they had to use $x$ to find the corresponding minimum value of $S$.

Part (d) was generally well attempted, even by those who did not complete Q10(c), with the majority remembering to compare $S^{\prime \prime}$ with 0 . However, an incorrect $S^{\prime \prime}$, which was quite common even for those with the correct $S^{\prime}$, did lose the final mark. A relatively small number of students set $S^{\prime \prime}=0$ and tried to solve for $x$, often concluding with $x>0 x>0$ so minimum.

## Statistics for C2 Practice Paper Silver Level S5

Mean score for students achieving grade:

| Qu | Max <br> score | Modal <br> score | Mean <br> $\%$ | ALL | A* $^{*}$ | A | B | C | D | E | $\mathbf{U}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | 4 |  | 81 | 3.22 |  | 3.71 | 3.42 | 3.06 | 2.65 | 2.39 | 1.64 |
| $\mathbf{2}$ | 7 | 5 | 67 | 4.70 | 6.79 | 6.22 | 5.47 | 4.97 | 4.40 | 3.77 | 2.45 |
| $\mathbf{3}$ | 9 | 9 | 71 | 6.42 | 8.86 | 8.53 | 7.87 | 7.10 | 6.04 | 4.67 | 1.88 |
| $\mathbf{4}$ | 8 |  | 71 | 5.69 |  | 7.30 | 6.20 | 5.29 | 4.32 | 3.53 | 2.23 |
| $\mathbf{5}$ | 9 | 9 | 66 | 5.95 | 8.76 | 8.24 | 7.17 | 6.19 | 5.19 | 4.11 | 2.29 |
| $\mathbf{6}$ | 13 |  | 60 | 7.79 | 12.44 | 11.31 | 8.80 | 6.72 | 4.49 | 3.25 | 1.32 |
| $\mathbf{7}$ | 11 |  | 59 | 6.46 | 9.96 | 9.17 | 7.70 | 6.64 | 5.60 | 4.28 | 1.84 |
| $\mathbf{8}$ | 14 |  | 58 | 8.07 | 13.27 | 12.51 | 10.43 | 8.30 | 6.14 | 4.22 | 1.80 |
|  | $\mathbf{7 5}$ |  | $\mathbf{6 4 . 4 0}$ | $\mathbf{4 8 . 3 0}$ | $\mathbf{6 0 . 0 8}$ | $\mathbf{6 6 . 9 9}$ | $\mathbf{5 7 . 0 6}$ | $\mathbf{4 8 . 2 7}$ | $\mathbf{3 8 . 8 3}$ | $\mathbf{3 0 . 2 2}$ | $\mathbf{1 5 . 4 5}$ |

